CALIFORNIA INSTITUTE OF TECHNOLOGY

Department of Electrical Engineering
Mail Code 136-93
Pasadena, CA 91125, USA

To Dr. Wen Masters **9/14/2015**
Office of Naval Research
875 North Randolph Street
Arlington, VA 22203-1995

Dear Dr. Masters:

It gives me great pleasure to send you my final report for the ONR grant N00014-11-1-0676. You had been kind enough to grant me a one year extension, and the project is now concluded. The report contains the following:

1. Complete list of publicaitons from the grant (31 May 2011 through 30 Sept. 2015).
2. A separate list of publications and invited talks during the final year (including extension). I have also included the abstracts of the journal papers in the final year, so that the description of our research is clear from these abstracts;

I also wanted to mention the following *prizes and recognitions* during the grant period.

1. I will be receiving the *IEEE Gustav Robert Kirchhoff Award,* to be presented in 2016 (a major IEEE Technical Field Award).
2. I received the *Education Award* of the IEEE Signal Processing Society, presented at the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP) 2012 (one of the major awards of the Signal Processing Society).
3. I am a coauthor of a paper for which the first author (Piya Pal, Ph.D student) received a student-paper prize: Piya Pal and P. P. Vaidyanathan, "Non Uniform Linear Arrays For Improved Identifiability in Cumulant Based DOA Estimation," Proc. 45th Asilomar Conference on Signals, Systems, and Computers, Monterey, CA, Nov. 2011.

Finally, here is the list of *graduate students* who benefited from support given by the grant. Some of them have already graduated and found excellent jobs:

1. Dr. Piya Pal (Ph. D 2013), now assitant professor at University of Maryland, College Park.
2. Dr. Chih-Hao Liu, (Ph. D 2013), now with Qualcom, San Diego.
3. Dr. Ching-Chih Weng, (Ph. D 2011), now with Facebook.

4. Srikanth Tenneti (current doctoral student)
5. Chun-Lin Liu (current doctoral student)

Once again, I want to convey my heartfelt thanks to you for making the transition smooth for me after Dr. Madan retired. The research work under this grant has been extremely rewarding for me and my graduate students. I sincerely hope that ONR has also benefitted from our work.

I am also mailing this report to all the addresses mentioned in the reporting instructions of the grant. Please do not hesitate to contact me for further information.

Sincerely,
P. P. Vaidyanathan

Prof. P. P. Vaidyanathan, Fellow, IEEE
Professor and past Department Head, Electrical Engineering, 136-93,
California Institute of Technology
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USA

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Publications under the grant N00014-11-1-0676

Full Period: 31 May 2011 through 30 Sept. 2015

Principal investigator: Prof. P. P. Vaidyanathan, California Institute of Technology

Invited plenaries in the above period

1) Plenary speaker at the International Conference on Computers and Devices for Communication (CODEC), Kolkata, India, December, 2015.
2) Plenary speaker at the Asia Pacific Signal and Information Processing Association (APSIPA) Annual Summit Conference, Siem Reap, Cambodia, December, 2014.
3) Plenary speaker at the National Symposium on Mathematical Methods and Applications, Chennai, India, Dec. 2013. This is held to honor the late Srinivasa Ramanujan every year.
4) Plenary speaker at the International Conf. on Comm. and Signal Processing (ICCSP), Calicut, India, 2011.

List of publications during the above period

Journal publications

1) P. P. Vaidyanathan and Piya Pal, "Theory of sparse coprime sensing in multiple dimensions", IEEE Trans. on Signal Processing, vol. 59, no. 8, August 2011.
2) P. P. Vaidyanathan and Piya Pal, "Generating new commuting coprime matrix pairs from known pairs", IEEE Signal Processing Letters, vol. 18, No. 5, pp. 303-306, May 2011.
3) P. P. Vaidyanathan and Piya Pal, "A general approach to coprime pairs of matrices, based on minors", IEEE Trans. on Signal Processing, vol. 59, no. 8, August 2011.
4) Chih-Hao Liu and P. P. Vaidyanathan, "Generalized Geometric Mean Decomposition and DFE Transceiver Design-Part I: Design and Complexity", IEEE Trans. on Signal Processing, vol. 60, No. 6, June 2012.
5) Chih-Hao Liu and P. P. Vaidyanathan, "Generalized Geometric Mean Decomposition and DFE Transceiver Design-Part II: Performance Analysis", IEEE Trans. on Signal Processing, vol. 60, No. 6, June 2012.
6) Ching-Chih Weng and P. P. Vaidyanathan, "The Role of GTD in Optimizing Perfect Reconstruction Filter Banks", IEEE Trans. on Signal Processing, vol. 60, No. 1, January 2012.
7) Piya Pal and P. P. Vaidyanathan, "Multiple Level Nested Array: An efficient geometry for 2qth order cumulant based array processing", IEEE Trans. on Signal Processing, vol. 60, No. 3, pp. 1253–1269, March 2012.
8) Piya Pal and P. P. Vaidyanathan, "Nested Arrays in Two Dimensions, Part I: Geometrical Considerations", IEEE Trans. on Signal Processing, vol. 60, No. 9, pp. 4694–4705, Sept. 2012.
9) Piya Pal and P. P. Vaidyanathan, "Nested Arrays in Two Dimensions, Part II: Application in Two Dimensional Array Processing", IEEE Trans. on Signal Processing, vol. 60, No. 9, pp. 4706–4718, Sept. 2012.
10) Chih-Hao Liu and P. P. Vaidyanathan, "MIMO Broadcast DFE Transceivers with QoS constraints: Min-Power and Max-Rate Solutions", IEEE Trans. on Signal Processing, vol. 61, no. 22, pp. 5550-5562, Nov. 2013.

11) Piya Pal and P. P. Vaidyanathan, "Pushing the Limits of Sparse Support Recovery Using Correlation Information", IEEE Trans. on Signal Processing, vol. 63, no. 3, pp. 711–726, Feb. 2015.
12) Piya Pal and P. P. Vaidyanathan, "A Grid-less approach to Underdetermined Direction of Arrival Estimation Via Low Rank Matrix Denoising", IEEE Signal Processing Letters, vol. 21, no. 6, pp. 737–741, June 2014.
13) P. P. Vaidyanathan, "Ramanujan sums in the context of signal processing: Part I: fundamentals," IEEE Trans. on Signal Proc., vol. 62, no. 16, pp. 4145–4157, Aug., 2014.
14) P. P. Vaidyanathan, "Ramanujan sums in the context of signal processing: Part II: FIR representations and applications," IEEE Trans. on Signal Proc., vol. 62, no. 16, pp. 4158–4172, Aug., 2014.
15) S. Tenneti and P. P. Vaidyanathan, "Nested Periodic Matrices and Dictionaries: New Signal Representations for Period Estimation," IEEE Trans. on Signal Proc., vol. 63, no. 14, pp. 3776-3790, July 2015.
16) S. Tenneti and P. P. Vaidyanathan, "Arbitrarily Shaped Periods in Multi-Dimensional Discrete Time Periodicity," IEEE Signal Processing Letters, vol. 22, no. 10, pp. 1748-1751, Oct. 2015.
17) C.-L. Liu and P. P. Vaidyanathan, "Remarks on the spatial smoothing step in coarray MUSIC," IEEE Signal Processing Letters, vol. 22, no. 9, pp. 1438–1442, Sept. 2015.
18) P. P. Vaidyanathan, "Ramanujan-sums in signal processing," Asia-Pacific Signal and Information Processing Association Newsletter, Issue 8, pp. 7–10, Jan. 2015.
19) Piya Pal and P. P. Vaidyanathan, "Limits of Sparse Support Recovery in Presence of Limited Cross Correlation", IEEE Trans. on Signal Processing, submitted, 2015.
20) S. Tenneti and P. P. Vaidyanathan, "A Unified Theory of Union of Subspaces Representations for Period Estimation," IEEE Trans. on Signal Proc., submitted.
21) O. Teke and P. P. Vaidyanathan, "Theory of Multirate Signal Processing on Graphs – Part I: Fundamentals," IEEE Trans. on Signal Proc., submitted.
22) O. Teke and P. P. Vaidyanathan, "Theory of Multirate Signal Processing on Graphs – Part II: M-Channel Filter Bank Theory," IEEE Trans. on Signal Proc., submitted.

Conference publications

23) P. P. Vaidyanathan and Piya Pal, "Coprime sampling for system stabilization with FIR multirate controllers," Proc. 45th Asilomar Conference on Signals, Systems, and Computers, Monterey, CA, Nov. 2011.
24) Piya Pal and P. P. Vaidyanathan, "Non Uniform Linear Arrays For Improved Identifiability in Cumulant Based DOA Estimation," Proc. 45th Asilomar Conference on Signals, Systems, and Computers, Monterey, CA, Nov. 2011.
25) C.-H Liu and P. P. Vaidyanathan, "Low Complexity Generalized Geometric Mean Decomposition and DFE Transceiver Design," Proc. 45th Asilomar Conference on Signals, Systems, and Computers, Monterey, CA, Nov. 2011.
26) C.-C Weng and P. P. Vaidyanathan, "Nonuniform Sparse Array Design for Active Sensing," Proc. 45th

Asilomar Conference on Signals, Systems, and Computers, Monterey, CA, Nov. 2011.

27) Piya Pal and P. P. Vaidyanathan, "Correlation-Aware Techniques for Sparse Support Recovery," 14th IEEE SSP workshop, Ann Arbor, MI, Aug. 2012.
28) C.-H Liu and P. P. Vaidyanathan, "MIMO Broadcast DFE Transceiver Design with Bit Allocation under QoS Constraints," 14th IEEE SSP workshop, Ann Arbor, MI, Aug. 2012.
29) P. P. Vaidyanathan and Piya Pal, "Direct-MUSIC on sparse arrays," IEEE Intl. Conf. on Signal Proc. and Comm., Bangalore, India, July 2012.
30) C.-H Liu and P. P. Vaidyanathan, "Max-Rate MIMO Broadcast DFE Transceiver Design under Power and SER Constraints," Proc. 46th Asilomar Conference on Signals, Systems, and Computers, Monterey, CA, Nov. 2012.
31) P. P. Vaidyanathan and Piya Pal, "Role of bandwidth in the quality of inversion of linear multirate systems with noise," Proc. 46th Asilomar Conference on Signals, Systems, and Computers, Monterey, CA, Nov. 2012.
32) Piya Pal and P. P. Vaidyanathan, "On Application of LASSO for Sparse Support Recovery With Imperfect Correlation Awareness," Proc. 46th Asilomar Conference on Signals, Systems, and Computers, Monterey, CA, Nov. 2012.
33) P. P. Vaidyanathan, "Compressive sensing and sparse array processing," Proc. 46th Asilomar Conference on Signals, Systems, and Computers, Monterey, CA, Nov. 2012.
34) Piya Pal and P. P. Vaidyanathan, "Correlation-aware sparse support recovery: Gaussian sources," Proc. IEEE Int. Conf. Acoust. Speech, and Signal Proc., Vancouver, Canada, May 2013.
35) P. P. Vaidyanathan, "Analog filter banks for sampling: discretization, polyphase form, and role in compressive sensing," 15th IEEE DSP workshop, Napa, CA, August 2013.
36) C.-C. Liu and P. P. Vaidyanathan, "Copilots in channel estimation," 15th IEEE DSP workshop, Napa, CA, August 2013.
37) Piya Pal and P. P. Vaidyanathan, "Conditions for Identifiability in Sparse Spatial Spectrum Sensing," European Signal Processing Conference," Marrakech, Morocco, September 2013.
38) P. P. Vaidyanathan and Piya Pal, "Why does direct-MUSIC on sparse-arrays work?," Proc. 47th Asilomar Conference on Signals, Systems, and Computers, Monterey, CA, Nov. 2013.
39) P. P. Vaidyanathan and Srikanth Tenneti, "Reflections on Sampling-Filters for Compressive Sensing and Finite-Innovations-Rate Models," Proc. 47th Asilomar Conference on Signals, Systems, and Computers, Monterey, CA, Nov. 2013.
40) P. P. Vaidyanathan, "Ramanujan-sum expansions for finite duration (FIR) sequences," Proc. IEEE Int. Conf. Acoust. Speech, and Signal Proc., Florence, Italy, May 2014.
41) P. P. Vaidyanathan and Piya Pal, "The Farey dictionary for sparse representation of periodic signals ," Proc. IEEE Int. Conf. Acoust. Speech, and Signal Proc., Florence, Italy, May 2014.
42) Piya Pal and P. P. Vaidyanathan, "Parameter identifiability in sparse Bayesian learning," Proc. IEEE Int. Conf.

Acoust. Speech, and Signal Proc., Florence, Italy, May 2014.

43) Piya Pal and P. P. Vaidyanathan, "Soft-Thresholding for Spectrum Sensing with Coprime Samplers," Proc. IEEE Sensor Array and Multichannel Signal Processing Workshop, Coruna, Spain, June 2014.
44) S. Tenneti and P. P. Vaidyanathan, "Dictionary approaches for identifying periodicities in data," Proc. Asil. Conf. Sig., Sys., and Comp., Monterey, CA, Nov. 2014.
45) P. P. Vaidyanathan and S. Tenneti, "Ramanujan subspaces and digital signal processing," Proc. Asil. Conf. Sig., Sys., and Comp., Monterey, CA, Nov. 2014.
46) C.-L. Liu and P. P. Vaidyanathan, "Design of coprime DFT arrays and filter banks," Proc. Asil. Conf. Sig., Sys., and Comp., Monterey, CA, Nov. 2014.
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48) P. P. Vaidyanathan, "Multidimensional Ramanujan-sum expansions on nonseparable lattices," Proc. IEEE Int. Conf. Acoust. Speech, and Signal Proc., Brisbane, April 2015.
49) S. Tenneti and P. P. Vaidyanathan, "Ramanujan filter banks for estimation and tracking of periodicity properties," Proc. IEEE Int. Conf. Acoust. Speech, and Signal Proc., Brisbane, April 2015.
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52) P. P. Vaidyanathan and S. Tenneti, "Properties of Ramanujan filter banks," Proc. European Signal Processing Conference (EUSIPCO), Nice, France, Sept. 2015.
53) S. Tenneti and P. P. Vaidyanathan, "Minimal Dictionaries For Spanning Periodic Signals," Proc. Asil. Conf. Sig., Sys., and Comp., Monterey, CA, Nov. 2015.
54) S. Tenneti and P. P. Vaidyanathan, "Period Estimation and Tracking: Filter Bank Design Using Truth Tables of Logic," Proc. Asil. Conf. Sig., Sys., and Comp., Monterey, CA, Nov. 2015.
55) C.-L. Liu and P. P. Vaidyanathan, "Tensor MUSIC in Multidimensional Sparse Arrays," Proc. Asil. Conf. Sig., Sys., and Comp., Monterey, CA, Nov. 2015.
56) O. Teke and P. P. Vaidyanathan, "Fundamentals of Multirate Graph Signal Processing," Proc. Asil. Conf. Sig., Sys., and Comp., Monterey, CA, Nov. 2015.

Publications under the grant N00014-11-1-0676

Period: 2013-15 (Final year plus extension).

Principal investigator: Prof. P. P. Vaidyanathan, California Institute of Technology

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1) Plenary speaker at the International Conference on Computers and Devices for Communication (CODEC), Kolkata, India, December, 2015.
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Journal publications

1) Chih-Hao Liu and P. P. Vaidyanathan, "MIMO Broadcast DFE Transceivers with QoS constraints: Min-Power and Max-Rate Solutions", IEEE Trans. on Signal Processing, vol. 61, no. 22, pp. 5550-5562, Nov. 2013.
2) Piya Pal and P. P. Vaidyanathan, "Pushing the Limits of Sparse Support Recovery Using Correlation Information", IEEE Trans. on Signal Processing, vol. 63, no. 3, pp. 711–726, Feb. 2015.
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Filter Bank Theory," IEEE Trans. on Signal Proc., submitted.

13) Piya Pal and P. P. Vaidyanathan, "Limits of Sparse Support Recovery in Presence of Limited Cross Correlation", IEEE Trans. on Signal Processing, in preparation, 2015.

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14) P. P. Vaidyanathan, "Analog filter banks for sampling: discretization, polyphase form, and role in compressive sensing," 15th IEEE DSP workshop, Napa, CA, August 2013.
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30) C.-L. Liu and P. P. Vaidyanathan, "Coprime DFT filter bank design: theoretical bounds and guarantees," Proc. IEEE Int. Conf. Acoust. Speech, and Signal Proc., Brisbane, April 2015.
31) P. P. Vaidyanathan and S. Tenneti, "Properties of Ramanujan filter banks," Proc. European Signal Processing Conference (EUSIPCO), Nice, France, Sept. 2015.
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35) O. Teke and P. P. Vaidyanathan, "Fundamentals of Multirate Graph Signal Processing," Proc. Asil. Conf. Sig., Sys., and Comp., Monterey, CA, Nov. 2015.

Abstracts for Journal publications

1) Chih-Hao Liu and P. P. Vaidyanathan, "MIMO Broadcast DFE Transceivers with QoS constraints: Min-Power and Max-Rate Solutions", IEEE Trans. on Signal Processing, vol. 61, no. 22, pp. 5550-5562, Nov. 2013.

   Abstract. This paper considers two joint design problems of linear precoder, decision feedback equalizer (DFE) and bit allocation for multi-input multi-output (MIMO) broadcast (BC) channels. The first problem is a power minimization problem (min-power) with a total bitrate constraint and per data stream symbol error rate (SER) specifications. The second problem is a rate maximization problem (max-rate) with a total transmit power constraint and per data stream SER specifications. For a given broadcast DFE transceiver, optimal bit allocation formulas for both problems are derived. A particular class of joint triangularization (JT) is applied to obtain the optimal broadcast DFE transceivers for the min-power and max-rate QoS problems, namely the minimum power JT broadcast DFE transceiver (MPJT) and the maximum rate JT broadcast DFE transceiver (MRJT), respectively. Two suboptimal broadcast DFE transceivers, the power minimized QR broadcast DFE transceiver (PMQR) and the rate maximized QR broadcast DFE transceiver (RMQR), are also proposed for the min-power and max-rate QoS problems, respectively. The proposed suboptimal designs apply QR decompositions instead of the particular class of JT. Moreover, integer bit allocation problems for both QoS problems are addressed. This work also shows the duality of the proposed MPJT and MRJT transceivers. At the end, numerical results are presented to demonstrate the performance of the proposed MPJT, MRJT, PMQR and RMQR transceivers under different QoS constraints, and verify the duality of the proposedMPJT and MRJT transceivers.

2) Piya Pal and P. P. Vaidyanathan, "Pushing the Limits of Sparse Support Recovery Using Correlation Information", IEEE Trans. on Signal Processing, vol. 63, no. 3, pp. 711–726, Feb. 2015.

   Abstract. A new framework for the problem of sparse support recovery is proposed, which exploits statistical information about the unknown sparse signal in the form of its correlation. A key contribution of this paper is to show that if existing algorithms can recover sparse support of size , then using such correlation information, the guaranteed size of recoverable support can be increased to $O(s^2)$, although the sparse signal itself may not be recoverable. This is proved to be possible by (a) formulating the sparse support recovery problem in terms of the covariance matrix of themeasurements, and (b) designing a suitablemeasurement/sampling matrix which inherently exploits the correlation priors. The so-called Khatri-Rao product of the measurement matrix is shown to play an important role in deciding the level of recoverable sparsity. A systematic analysis of the proposed framework is also presented for the cases when the covariance matrix is only approximately known, by estimating it from finite number of measurements, obtained from the Multiple Measurement Vector (MMV) model. In this case, the use of LASSO on the estimated covariance matrix is proposed for recovering the support. However, the recovery may not be exact and hence a probabilistic guarantee is developed both for sources with arbitrary distribution as well as for Gaussian sources. In the latter case, it is shown that such recovery can happen with overwhelming probability as the number of available measurement vectors increases.

3) Piya Pal and P. P. Vaidyanathan, "A Grid-less approach to Underdetermined Direction of Arrival Estimation Via Low Rank Matrix Denoising", IEEE Signal Processing Letters, vol. 21, no. 6, pp. 737–741, June 2014.

   Abstract. The problem of direction of arrival (DOA) estimation of narrowband sources using an antenna array is considered where the number of sources can potentially exceed the number of sensors. In earlier works, the authors showed that using a suitable antenna geometry, such as the nested and coprime arrays, it is possible to localize $O(M^2)$ sources using $O(M)$ sensors. To this end, two different approaches have been proposed. One is based on an extension of subspace based methods such as MUSIC to these sparse arrays, and the other employs ℓ_1 norm minimization based sparse estimation techniques by assuming an underlying grid. While the former requires the knowledge of number of sources, the latter suffers from basis mismatch effects. In this letter, a new approach is proposed which overcomes both these weaknesses. The method is hybrid in nature, using a low rank matrix denoising approach followed by a MUSIC-like subspace method to estimate the DOAs. The number of sources is revealed as a by-product of the low rank denoising stage. Moreover, it does not assume any underlying grid and thereby does not suffer frombasis mismatch. Numerical examples validate the effectiveness of the proposed method when compared against existing techniques.

4) P. P. Vaidyanathan, "Ramanujan sums in the context of signal processing: Part I: fundamentals," IEEE Trans. on Signal Proc., vol. 62, no. 16, pp. 4145–4157, Aug., 2014.

   Abstract. The famousmathematician S. Ramanujan introduced a summation in 1918, now known as the Ramanujan sum $c_q(n)$. For any fixed integer , this is a sequence in with periodicity q. Ramanujan showed that many standard arithmetic functions in the theory of numbers, such as Eulers totient function $\phi(n)$ and the Mobius function , can be expressed as linear combinations of $c_q(n), 1 \leq q \leq \infty$. In the last ten years, Ramanujan sums have aroused some interest in signal processing. There is evidence that these sums can be used to extract periodic components in discrete-time signals. The purpose of this paper and the companion paper (Part II) is to develop this theory in detail. After a brief review of the properties of Ramanujan sums, the paper introduces a subspace called the Ramanujan subspace and studies its properties in detail. For fixed , the subspace includes an entire family of signals with properties similar to $c_q(n)$. These subspaces have a simple integer basis defined in terms of the Ramanujan sum $c_q(n)$ and its circular shifts. The projection of arbitrary signals onto these subspaces can be calculated using only integer operations. Linear combinations of signals belonging to two or more such subspaces follows certain specific periodicity patterns, which makes it easy to identify periods. In the companion paper (Part II), it is shown that arbitrary finite duration signals can be decomposed into a finite sum of orthogonal projections onto Ramanujan subspaces.

5) P. P. Vaidyanathan, "Ramanujan sums in the context of signal processing: Part II: FIR representations and applications," IEEE Trans. on Signal Proc., vol. 62, no. 16, pp. 4158–4172, Aug., 2014.

Abstract. The mathematician Ramanujan introduced a summation in 1918, now known as the Ramanujan sum $c_q(n)$. In a companion paper many properties of Ramanujan sums were reviewed, and Ramanujan subspaces $\mathcal{S}_q$ were introduced, of which the Ramanujan sum is a member. The properties of signals belonging to $\mathcal{S}_q$ were studied in detail. In this paper the problem of representing finite duration (FIR) signals based on Ramanujan sums and spaces is considered. First it is shown that the traditional way to solve for the expansion coefficients in the Ramanujan-sum expansion does not work in the FIR case. Two solutions are then developed. The first one is based on a linear combination of the first N Ramanujan-sums (where N is also the length of the FIR signal). It is shown that this method is not useful for identifying periods or hidden periods in the sequence. The second solution is based on the use of Ramanujan subspaces. With $q_1, q_2, \ldots, q_K$ denoting the divisors of N, it is shown that $x(n)$ can be written as a sum of K signals $x_{q_i}(n) \in S_{q_i}$. Furthermore the ith signal $x_{q_i}(n)$ has period q_i, and any pair of these periodic components is orthogonal. The components $x_{q_i}(n)$ can be calculated by finding the orthogonal projections of $x(n)$ onto the Ramanujan spaces $\mathcal{S}_{q_i}$. It is also shown that if the signal $x(n)$ has a periodicity $N_x < N$, then it is possible to identify this period by analyzing the components $x_{q_i}(n)$. A transform called the Ramanujan Periodic Transform (RPT) is defined based on this. The Ramanujan spaces $\mathcal{S}_{q_i}$ can be represented either with an integer basis (based on $c_{q_i}(n)$ and its circular shifts) or a Vandermonde basis (based on columns of DFT matrices). The former basis can be used to show that the projection matrices (which compute $x_{q_i}(n)$ from $x(n)$) are integer matrices except for an overall scale factor. The calculation of projections is rendered easy because of this. In order to estimate any possible internal period $N_x < N$ of $x(n)$, one only needs to know which of the projection energies are nonzero. The paper also shows how to reduce the effect of noise by performing projections based on signal correlations.

6) S. Tenneti and P. P. Vaidyanathan, "Nested Periodic Matrices and Dictionaries: New Signal Representations

for Period Estimation," IEEE Trans. on Signal Proc., vol. 63, no. 14, pp. 3776-3790, July 2015.

Abstract. AbstractIn this paper, we propose a new class of techniques to identify periodicities in data. We target the period estimation directly rather than inferring the period from the signals spectrum. By doing so, we obtain several advantages over the traditional spectrum estimation techniques such as DFT and MUSIC. Apart from estimating the unknown period of a signal, we search for finer periodic structure within the given signal. For instance, it might be possible that the given periodic signal was actually a sum of signals with much smaller periods. For example, adding signals with periods 3, 7, and 11 can give rise to a period 231 signal. We propose methods to identify these hidden periods 3, 7, and 11. We first propose a new family of square matrices called Nested Periodic Matrices (NPMs), having several useful properties in the context of periodicity. These include the DFT, WalshHadamard, and Ramanujan periodicity transform matrices as examples. Based on these matrices, we develop high dimensional dictionary representations for periodic signals. Various optimization problems can be formulated to identify the periods of signals from such representations. We propose an approach based on finding the least ℓ_2 norm solution to an under-determined linear system. Alternatively, the period identification problem can also be formulated as a sparse vector recovery problem and we show that by a slight modification to the usual ℓ_1 norm minimization techniques, we can incorporate a number of new and computationally simple dictionaries.

7) S. Tenneti and P. P. Vaidyanathan, "Arbitrarily Shaped Periods in Multi-Dimensional Discrete Time Periodicity," IEEE Signal Processing Letters, vol. 22, no. 10, pp. 1748-1751, Oct. 2015.

Abstract. Traditionally, most of the analysis of discrete time multidimensional periodicity in DSP is based on defining the period as a parallelepiped. In this work, we study whether this framework can incorporate signals that are repetitions of more general shapes than parallelepipeds. For example, the famous Dutch artist M. C. Escher constructed many interesting shapes such as fishes, birds and animals, which can tile the continuous 2-D plane. Inspired from Eschers tilings, we construct discrete time signals that are repetitions of various kinds of shapes. We look at periodicity in the following way - a given shape repeating itself along fixed directions to tile the entire space. By transcribing this idea into a mathematical framework, we explore its relationship with the traditional analysis of periodicity based on parallelepipeds. Our main result is that given any such signal with an arbitrarily shaped period, we can always find an equivalent parallelepiped shaped period that has the same number of points as the original period.

8) C.-L. Liu and P. P. Vaidyanathan, "Remarks on the spatial smoothing step in coarray MUSIC," IEEE Signal Processing Letters, vol. 22, no. 9, pp. 1438–1442, Sept. 2015.

Abstract. Sparse arrays such as nested and coprime arrays use a technique called spatial smoothing in order to successfully perform MUSIC in the difference-coarray domain. In this paper it is shown that the spatial smoothing step is not necessary in the sense that the effect achieved by that step can be obtained more directly. In particular, with $\widetilde{\mathbf{R}}_{ss}$ denoting the spatial smoothed matrix with finite snapshots, it is shown here that the noise eigenspace of this matrix can be directly obtained from another matrix $\widetilde{\mathbf{R}}$ which is much easier

to compute from data.

9) P. P. Vaidyanathan, "Ramanujan-sums in signal processing," Asia-Pacific Signal and Information Processing Association Newsletter, Issue 8, pp. 7–10, Jan. 2015.
10) S. Tenneti and P. P. Vaidyanathan, "A Unified Theory of Union of Subspaces Representations for Period Estimation," IEEE Trans. on Signal Proc., submitted.

 Abstract. A number of well-known period estimation techniques can be interpreted as being based on recovering a signals support from a union of subspaces representing different periods. However, most of these techniques emerged independent of each other from diverse mathematical contexts, and so they propose and analyze only specic examples of such subspaces. This paper derives (i) the relationships between many such techniques under one unifying framework, and (ii) a number of their fundamental properties. For example, it is shown that the previously proposed Exactly Periodic Subspaces and the Ramanujan Subspaces are in fact the same, and are special cases of the more general Nested Periodic Subspaces (NPS). Furthermore, it is shown that the NPSs themselves emerge as natural candidates if one desires uniqueness in such subspace representations. Unique decompositions offer better accuracy and signicantly lower computations during period estimation. In the original framework, the NPSs had their dimensions chosen according to the Euler totient function in an ad hoc fashion. Doing so offered several useful properties such as the LCM property. In this work, it is shown that the Euler-structure, the LCM property and all other features of the NPSs are in fact fundamental consequences of requiring unique representations of periodic signals. One way to use such union of subspaces models for period estimation is to construct dictionaries. We derive the fundamental properties of such dictionaries such as their minimum required size, composition, and so on, in addition to discussing the various possible recovery techniques based on such dictionaries.
11) O. Teke and P. P. Vaidyanathan, "Theory of Multirate Signal Processing on Graphs – Part I: Fundamentals," IEEE Trans. on Signal Proc., submitted.

 Abstract. Signal processing on graphs finds applications in many areas. In recent years renewed interest on this topic was kindled by two groups of researchers. Narang and Ortega constructed two-channel filter banks on bipartitie graphs described by Laplacians. Sandryhaila and Moura developed the theory of linear systems, filtering, and frequency responses for the case of graphs with arbitrary adjacency matrices, and showed applications in signal compression, prediction, etc. Inspired by these contributions, this paper develops the complete theory of multirate systems for graph signals. The graphs are assumed to be general with a possibly non-symmetric and complex adjacency matrix. The paper revisits ideas such as noble identities, aliasing, and polyphase decompositions in graph multirate systems. It is shown that the extension of classical multirate theory to graphs is nontrivial, and requires certain mathematical restrictions on the graph. For example, classical noble identities cannot be taken for granted, and require restrictions on the adjacency matrix $\mathbf{A}$. Similarly, one cannot claim that thè so-called delay chain system is a perfect reconstruction system (as in classical filter banks). It will also be shown that M-partitie extensions of the bipartite filter bank results will not work for M-channel filter banks, but a more restrictive condition called M-block cyclic property should be imposed. Such graphs are studied in detail. The concept of spectrum folding (aliasing) is developed for such graphs. These results are used to develop a detailed theory for M-channel filter banks in the companion paper (Part II).

12) O. Teke and P. P. Vaidyanathan, "Theory of Multirate Signal Processing on Graphs – Part II: M-Channel Filter Bank Theory," IEEE Trans. on Signal Proc., submitted.

   Abstract. This paper builds upon the basic theory of multirate systems for graph signals developed in the companion paper (Part I), and studies M-channel polynomial filter banks on graphs. The behavior of such graph filter banks differs from that of classical filter banks in many ways, the precise details depending on the eigenstructure of the adjacency matrix $\mathbf{A}$. It is shown that graph filter banks represent (linear and) periodically shift variant systems only when A satisfies the noble identity conditions developed in Part I. It is then shown that perfect reconstruction graph filter banks can always be developed when A satisfies the eigenvector structure satisfied by M-block cyclic graphs and has distinct eigenvalues (eigenvalue restrictions being unnecessary for this). If $\mathbf{A}$ is actually M-block cyclic then these PR filter banks indeed become practical, i.e., arbitrary filter polynomial orders are possible, and there are robustness advantages. In this case the PR condition is identical to PR in classical filter banks any classical PR example can be converted to a graph PR filter bank on an M-block cyclic graph. It is shown that for M-block cyclic graphs with all eigenvalues on the unit circle, the frequency responses of filters have meaningful correspondence with classical filter banks. Polyphase representations are then developed for graph filter banks, and utilized to develop alternate conditions for alias cancellation and perfect reconstruction, again for graphs with specific eigenstructures. It is then shown that the eigenvector condition on the graph can be relaxed by using similarity transforms.

13) Piya Pal and P. P. Vaidyanathan, "Limits of Sparse Support Recovery in Presence of Limited Cross Correlation", IEEE Trans. on Signal Processing, in preparation, 2015.